Statistics
Summer 2023
Lecture 16

Feb 19-8:47 AM

Testing One Population Proportion: SG 25

$H_0: P = P_0$ $H_0: P \le P_0$ $H_0: P \ge P_0$

$H_1: P \ne P_0$ $H_1: P > P_0$ $H_1: P < P_0$

TTT RTT LTT

Always identify the claim

Find all critical values

Drawing, labeling, shading, TI command invNorm

Find CTS Z & P-Value P

1-PropZTest, $Z = \dfrac{\hat{P} - P}{\sqrt{\frac{PQ}{n}}}$

use normalcdf

use Testing Chart to determine the validity of H_0 & H_1

Draw final conclusion about the claim

Reject the claim OR **FTR the claim**

claim is invalid claim is valid

Jul 10-9:34 AM

The College claims that 20% of all students are in favor of online classes. $p = .2$ claim

I surveyed 150 students and 28% of them were in favor of online classes. $n = 150$, $\hat{p} = .28$

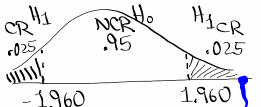
Use this survey to test the claim. $x = n\hat{p} = 150(.28)$

No α $\Rightarrow$ use .05 $\quad x = 42$

H_0: $p = .2$ claim

H_1: $p \neq .2$ TTT

CV z TTT $\alpha = .05$



CTS $z = 2.449$

P-value $p = .014$ ✓

1-PropZTest

$p_0 = .2$ H_0

$x = 42$

$n = 150$

Prop. $\neq p_0$ H_1

Calculate

$z = \text{invNorm}(.975, 0, 1)$

CTS is in CR $\Rightarrow$ H_0 invalid

P-value $\leq \alpha$ $\Rightarrow$ H_1 valid

.014 .05

Invalid claim $\Rightarrow$ Reject the claim

If we change α to .01, then

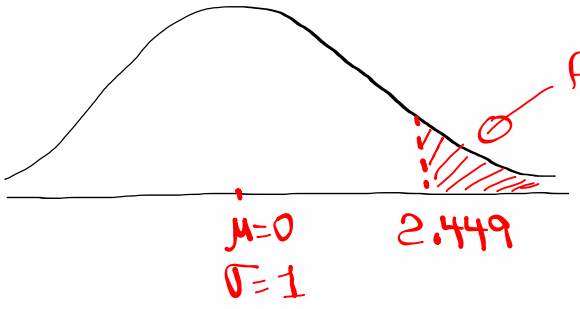
P-value $> \alpha$ $\Rightarrow$ H_0 valid $\Rightarrow$ valid claim

.014 > .01 H_1 invalid

FTR the claim

Jul 11-7:36 AM

Given CTS $z = 2.449$, TTT, find P-value.



Area * 2 = P-value

P-Value = 2 * normalcdf(2.449, E99, 0, 1)

= .014

Jul 11-7:48 AM

The college claims that less than 10% of all female students are STEM majors.

$p < .1$ claim

I surveyed 175 female students and 8.5% of them were STEM majors.

$n = 175$, $\hat{p} = .085$

$x = n\hat{p} = 175(.085) = 14.875$

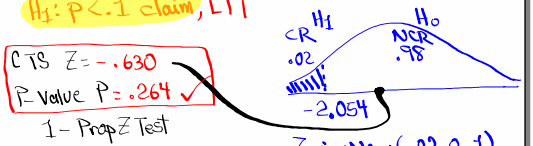
$x = 15$

Use $\alpha = .02$ to test the claim.

$H_0: p \geq .1$

$H_1: p < .1$ claim, LTT

CV z LTT $\alpha = .02$



CTS $z = -.630$

P-value $p = .264$ ✓

1-PropZTest

$p_0: .1$ H_0

$x = 15$

$n = 175$

Prop < p_0 H_1

Calculate

$z = \text{invNorm}(.02, 0, 1)$

CTS is in NCR. ⇒ H_0 valid, H_1 invalid

P-value $> \alpha$

$.264 > .02$

Invalid claim ⇒ Reject the claim

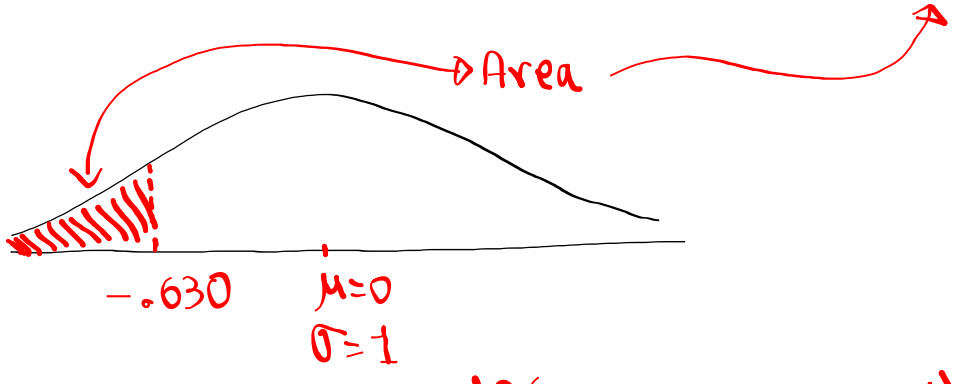
Suggest a value for α to reverse the conclusion.

P-value $\leq \alpha$

$.264 \leq \alpha$ ⇒ Pick $\alpha = .27, .28, .29, .3, \ldots$

Jul 11-7:52 AM

Given CTS $z = -.630$, LTT, find P-value.



P-value = normalcdf(−E99, −.630, 0, 1)

= .264

Jul 11-8:04 AM

Testing One Population Mean:

SG 26

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$ — TTT

$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$ — LTT

$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$ — RTT

Always identify the claim

Case I: σ known	Case II: σ unknown
CV z invNorm	CV t invT $df = n-1$
Drawing, labeling, shading, and TI command.	Drawing, labeling, shading, and TI command.
CTS z P-value p ⇒ Z-Test Input: Stats	CTS t P-value p ⇒ T-Test Input: Stats

We proceed with testing chart to learn about the validity of H_0 & H_1.

Draw Final Conclusion about the claim

Reject the claim when claim is invalid **OR FTR the claim** when claim is valid

Jul 10-10:45 AM

LA Times claim that the mean age of all teachers in LAUSD is 50 years. $\mu = 50$ claim H_0

I randomly selected 28 teachers from LAUSD, and their mean age was 45 years. $n = 28$ $\bar{x} = 45$

It is known that standard deviation of ages of all teachers in LAUSD is 7.5 yrs. $\sigma = 7.5$

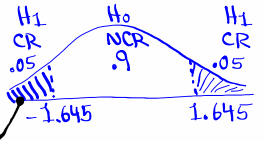
Use $\alpha = .1$ to test the claim.

$H_0: \mu = 50$ claim

$H_1: \mu \neq 50$ TTT

Since σ is known

CV z TTT $\alpha = .1$



CTS $z = -3.528$

P-value $p = 4.19 \times 10^{-4}$

Z-Test

Input: Stats

$\mu_0 = 50$ H_0

$\sigma = 7.5$

$\bar{x} = 45$

$n = 28$

$\mu \neq \mu_0$ H_1

$Z = \text{invNorm}(.95, 0, 1)$

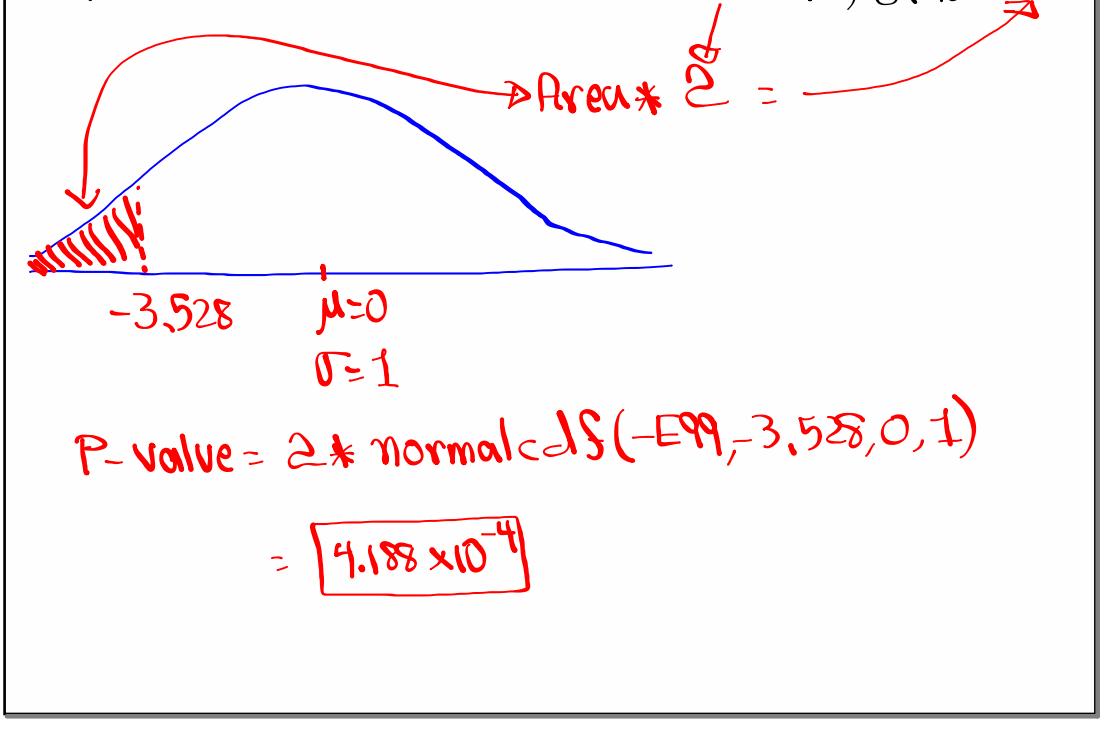
CTS is in CR ⇒ H_0 invalid

P-value $\leq \alpha$ ⇒ H_1 valid

Invalid claim ⇒ **Reject the claim**

Jul 11-8:26 AM

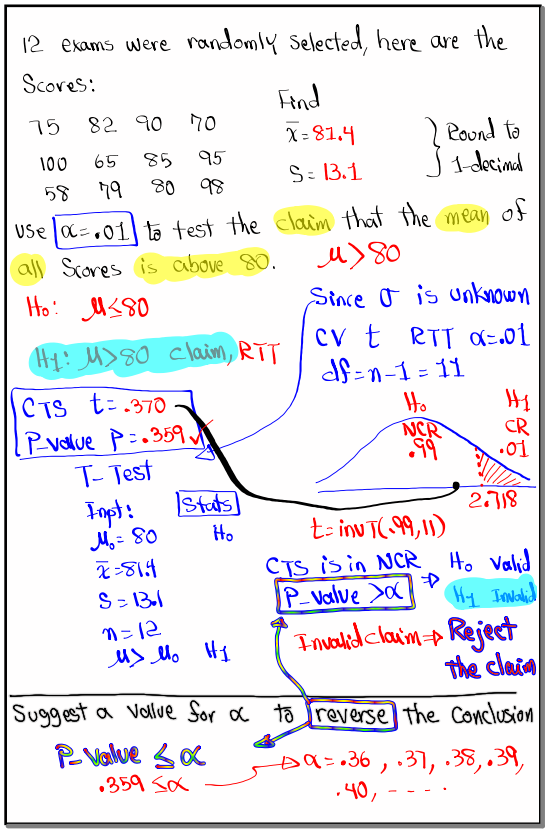
Given CTS $Z = -3.528$, TTT, Find P-value



P-value = 2 * normalcdf(-E99, -3.528, 0, 1)

= 4.188×10^{-4}

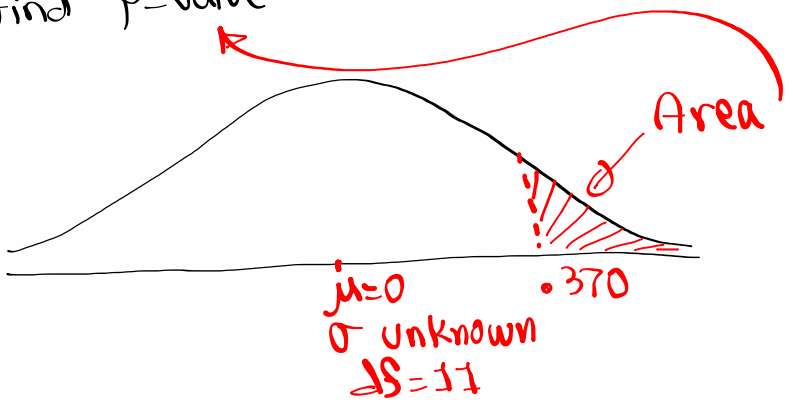
Jul 11-8:38 AM



Jul 11-8:41 AM

Given CTS $t = .370$, $df = 11$, RTT

Find P_value



P_value = tcdf(.370, E99, 11): .359

Jul 11-8:55 AM

8 randomly selected nurses had a mean salary of \$6400/mo. with standard deviation of \$500.

Test the claim that the mean salary for all nurses is below \$6750

$n = 8$
$\bar{x} = 6400$
$s = 500$

$\mu < 6750$

No $\alpha \Rightarrow$ use .05

H_0: $\mu \geq 6750$

H_1: $\mu < 6750$ claim, LTT

σ is unknown

CV t LTT $\alpha = .05$

$df = n - 1 = 8 - 1 = 7$

CTS $t = -1.980$

P_value $P = .044$ ✓

T-Test

Inpt: Stats

$\mu_0 = 6750$ H_0

$\bar{x} = 6400$

$s = 500$

$n = 8$

$\mu < \mu_0$ H_1

CR .05 H_1

NCR .95 H_0

-1.895

t=invT(.05,7)

CTS is in CR H_0 invalid

P_value $\leq \alpha \Rightarrow$ H_1 valid

valid claim $\Rightarrow$ FTR the claim

Suggest a value for α to reverse the Conclusion.

P_value $> \alpha$

$.044 > \alpha$

choose α to be .043, .04, 0.03, .02, .01

Jul 11-8:58 AM

χ^2 - Dist.

Chi - Sqr dist.

It comes with df

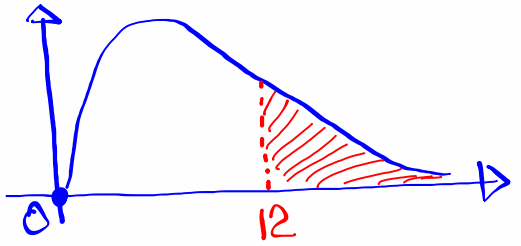
Total Area = 1

use $\chi^2cdf(L, U, df)$

Find $P(\chi^2 > 12)$ with $df = 9$.

$= \chi^2cdf(12, E99, 9)$

$= .213$

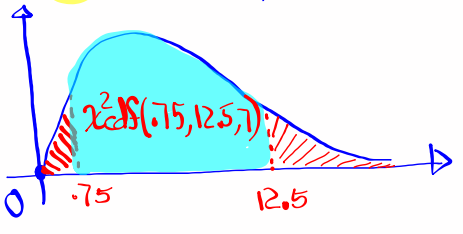


Jul 11-9:44 AM

Find $P(\chi^2 < 1.725)$ with $df = 8$.

$= \chi^2cdf(0, 1.725, 8) = .012$

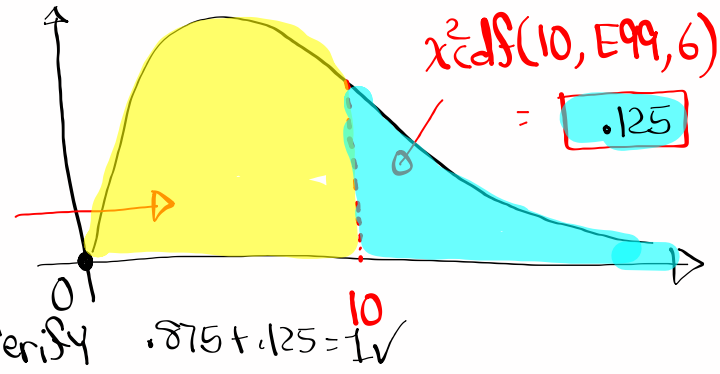
Find $P(\chi^2 < .75$ OR $\chi^2 > 12.5)$ with $df = 7$



$= 1 - \chi^2cdf(.75, 12.5, 7) = .087$

Jul 11-9:47 AM

Find the area on each side of $\chi^2 = 10$ with df = 6, then multiply the smaller area by 2.



2 * Smaller area = 2(.125) = .25

SG 27

Jul 11-9:52 AM

Testing One Population standard deviation

$H_0: \sigma = \sigma_0$	$H_0: \sigma \leq \sigma_0$	$H_0: \sigma \geq \sigma_0$
$H_1: \sigma_1 \neq \sigma_0$	$H_1: \sigma > \sigma_0$	$H_1: \sigma < \sigma_0$
TTT	RTT	LTT

Always identify the claim

Use P-value method only

CTS Formula $\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$

Use χ^2cdf with df = n − 1 to find P-value

Proceed with testing chart

Draw final conclusion about the claim

Jul 11-9:56 AM

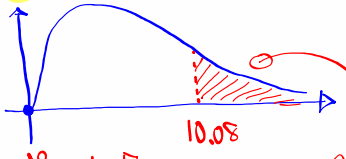
Given $n=8$, $S=12$, $H_1: \sigma > 10$, $\alpha = .02$

claim is H_1.

Test the claim.

$H_0: \sigma \leq 10$

$H_1: \sigma > 10$ claim, RTT



$df = n-1 = 7$

CTS

$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(8-1)\cdot 12^2}{10^2} = 10.08$

Area = P-Value

$= \chi^2 cdf(10.08, E99, 7)$

$= .184$

P-Value $>$ α

.184 .02

H_0 valid

H_1 invalid $\Rightarrow$ Invalid claim $\Rightarrow$ Reject the claim

If we choose $\alpha = .19, .20, .21, .22, \ldots$

P-Value $\leq \alpha \rightarrow H_1$ valid $\Rightarrow$ Valid claim

FTR the claim

Jul 11-10:02 AM

College claims that standard deviation of ages of all students is below 10 yrs.

$\sigma < 10$ claim

In a survey of 15 students, the standard deviation of their ages was 8 yrs.

$n=15$, $S=8$

Test the claim at $\alpha = .1$

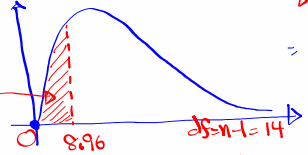
$H_0: \sigma \geq 10$

$H_1: \sigma < 10$ claim, LTT

CTS

$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} \rightarrow \frac{(15-1)\cdot 8^2}{10^2}$

$= 8.96$



Area = P-Value = $\chi^2 cdf(0, 8.96, 14) = .166$

P-Value $>$ α

.166 .1

H_0 valid H_1 invalid

Invalid claim

Reject the claim

Suggest a value for α To reverse the conclusion.

P-Value $\leq \alpha$

$.166 \leq \alpha$

Pick α to be $.17, .18, .19, .2, .21, .22, \ldots$

Jul 11-10:11 AM

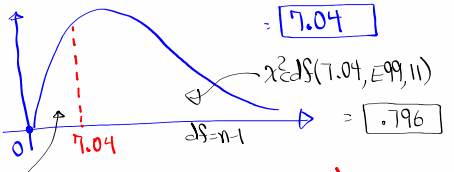
LA Times claims that standard deviation of salaries of all nurses is $500.
$\sigma = 500$ H_0, claim

I took a sample of 12 nurses, standard deviation of their salaries was $400.
$n = 12$, $s = 400$

Test the claim. ⇒ No α → use .05

H_0: $\sigma = 500$ claim
H_1: $\sigma \neq 500$ TTT

CTS
$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2} = \frac{(12-1)\cdot 400^2}{500^2}$
$= 7.04$



For TTT: Find area on each side
Multiply smaller area by 2.

χ^2cdf(0, 7.04, 11) = .204 Notice .204 + .796 = 1 ✓

P-value = 2 * Smaller area = 2 (.204) = .408

P-value > α ⇒ H_0 valid → valid claim
.408 > .05 H_1 invalid FTR the claim

Jul 11-10:22 AM

Below are Test Scores for randomly selected exams

78	65	100	90
80	85	70	58
	95	95	

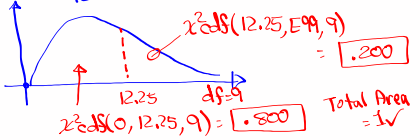
Find
1) $\bar{x} = 82$
2) $s = 14$
} Round to whole #
3) $n = 10$

No α → use .05

Test the claim that standard deviation of all scores is 12.

H_0: $\sigma = 12$ claim
H_1: $\sigma \neq 12$ TTT

CTS
$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$
$= \frac{(10-1)\cdot 14^2}{12^2} = 12.25$



Total Area = 1 ✓

P-value = 2 * Smaller area
= 2 (.200) = .400

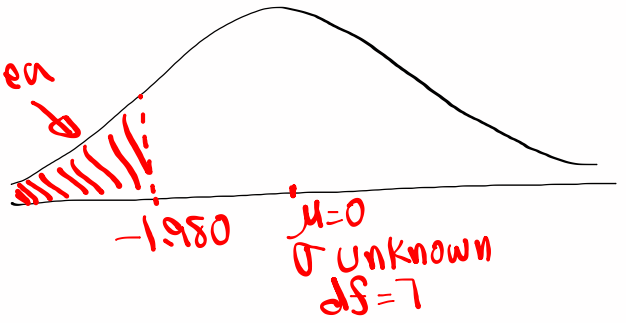
P-value > α H_0 valid → valid claim
.400 > .05 H_1 invalid FTR the claim

SG 27

Jul 11-10:34 AM

Given CTS $t = -1.980$, LTT, $df = 7$

Find P-value.



P-value = tcdf(-E99, -1.980, 7) = .044

Jul 11-9:12 AM